When do higher disease prevalences lead to increased demand for prevention: using a game theoretic model to explore the prevalence elasticity of prevention?

Rodney Beard and Louise Matthews
Institute for Biodiversity, Animal Health and Comparative Medicine
and the Boyd Orr Centre for Population and Ecosystem Health
University of Glasgow
Rodney.M.Beard@gmail.com
http://rbeard.sdf.org

Background

As a disease spreads individual prevention decisions may may impact the rate at which the disease spreads. How do rational individuals impact disease dynamics in the context of livestock diseases?

Literature

Early work on optimisation models in epidemiology

- stochastic dynamic programming (Abakkuk and others early 1970's)
- optimal control theory Wickwire (early to mid 1970's)
- Models concentrated on minimising costs of controlling a disease

Economic epidemiology

- combines economics with epidemiology
- initiated by Tomas Philipson and Richard Posner 1993
- Key idea 1: prevention depends on prevalence
- prevention can include altering contact behaviour, vaccination, isolation, treatment
- Key idea 2: prevalence elasticity of prevention (percentage change in prevention due to a percentage change in prevalence
- diseases can be induced, exacerbated or prevented through rational self-interested behaviour

Prevalence elasticity of prevention

As disease spreads if prevalence elasticity of prevention is positive then prevention increases slowing spread of disease. If the prevalence elasticity of prevention is negative then as the disease spreads prevention is reduced resulting in a faster spread of the disease. Latter idea is captured by "rational fatalism"

Some background literature

- Brito, Sheshinski and Intriligator (1991) JPubE show that free choice in vaccination is preferable to compulsory vaccination and yet is not socially optimal.
- Geoffard and Philipson (1996) IER consider the case of rational epidemics-epidemics induced by rational betaviour
- Geoffard and Philipson (1997) AER study a steady-state SIR model of private versus public vaccination
- a number of similar albeit fewer studies by epidemiologists published in leading science journals
- Many other papers in the emerging field of economic epidemiology as well.

Economic epidemiology and livestock diseases

- Bicknell, Wilen and Howitt (1999) AJARE (no strategic behaviour)
- Ceddia (2012) ERE solves open-loop differential game with SIS dynamics.
- Skonhoft and Sikhweni (recent unpublished working paper) applied to wildlife and livestock.

Optimal disease eradication

- Barrett (2003) JEEA, Barrett and Hoel (2007) EDE
- Under what conditions can a disease be eradicated
- Similar arguments apply for disease outbreak
- Can disease be eradicated in finite time?
- They approximate disease dynamics with a linear (Harmonic) oscillator
- They study a variety of cost specification but ignore health benefits

Epidemiology-Revision

$$\dot{S} = \mu N - \beta S I - \mu I$$

$$\dot{I} = \beta S I - \gamma I - \mu I$$

$$\dot{R} = \gamma I - \mu R$$

Let $\lambda = \beta I$ be the force of infection, then $\dot{I} = \frac{\dot{\lambda}}{\beta}$, $R_0 = \frac{\beta N}{\mu + \gamma}$
Substituting and rearranging we get

$$\dot{S} = \mu N - (\lambda + \mu) S$$

$$\dot{\lambda} = (\mu + \gamma)\lambda \left[R_0 S - 1\right]$$

$$\dot{R} = \frac{\gamma R_0 (\mu + \gamma)\lambda}{N} - \mu R$$

Now set $N = 1$!

Policy and Behavioural variables

What are possible policy or behavioural variables?

Vaccination (prevention)
Treatment of the ill
Transmission rate might change

Controlling disease through prevention

$$\dot{S} = \mu - (\lambda + \mu)S - p$$
$$\dot{\lambda} = (\mu + \gamma)\lambda \left[R_0 S - 1\right]$$

where p is some level of prevention.

How should p be chosen?

- choose p to eradicate disease?
- However this is costly-perhaps minimise costs and choose p accordingly
- However this ignores health benefits which might depend on the level of prevention
- Who should choose p?
- The state or the individual?

Optimal choice of p by a social planner?

$$\max_{p} \int_{0}^{\infty} e^{-\rho t} [B(S) - C(\lambda, p)] \, dt$$

subject to

$$\dot{S} = \mu - (\lambda + \mu)S - p$$

$$\dot{\lambda} = (\mu + \gamma)\lambda [R_0 S - 1]$$

$$S(0) = S_0, \lambda(0) = \lambda_0$$

Note! There is only ONE decision-maker! Barrett and Hoel solve this with $B(S) = 0$.

Differential games

- First studied by Isaacs (1940's and 1950's)
- N-player non-cooperative differential games first analysed by Case (1967) PhD Thesis Michigan and Starr and Ho (1969) JOTA

Information structures and solutions

There are in general two possible ways to solve linear quadratic differential games:

- open-loop solution (solution depends on initial conditions and time only-precommitment to strategies at beginning of play)
  - solved using Pontryagin's maximum principle
- feedback solution (leading to state dependent Markov perfect equilibria) -in the mathematics literature this is sometimes called closed-loop no memory
  - solved using continuous-time dynamic programming-solve Hamilton-jacobi-Bellman equation
  - method involves conjecturing a candidate value function (potential function) approximatted as a Taylor expansion
  - then solving for undetermined coefficients -this is the Riccati method because it leads to a system of Riccati (quadratic polynomial) differential equations

Differential prevention game

$$V^i(I,p,t) = \max_{\{p_i(t)\}} \int_0^{\infty} e^{-\rho t} \left\{ r(1-I) - \frac{c_i}{2} p_i^2 - \frac{b}{2} I^2 \right\} dt \; i = 1, \ldots, n$$

$$\dot{I} = \sigma \left[I^*(\sum_{i=1}^{n} p_i) - I \right]$$

where I^* is the steady-state level of I. Follows Barrett and Hoel (2007)). Where the steady-state from the Anderson and May system is

$$I^* = \frac{R_0}{\beta} \left(\mu(1 - \frac{1}{R_0}) - \sum_{i=1}^{n} p_i \right)$$

.

Open-loop Nash equilibrium

Definition

Open-loop Nash Equilibrium A strategy profile $(p_1^*, \ldots, p_n^*)$ is an open loop Nash equilibrium id
$V^i(p_1^*, \ldots, p_n^*) \geq V^i(p_1^*, \ldots, , p_i, \ldots, p_n^*)$ and $p_i(t) = \psi(t), \forall i$.

Differential prevention game

Each player therefore solves the following problem:

$$\max_{\{p_i(t)\}} \int_0^\infty e^{-\rho t} \left\{ r(1-I) - \frac{c_i}{2} p_i^2 - \frac{b}{2} I^2 \right\} dt \; i = 1, \ldots, n$$

$$\dot{I} = \sigma \left[\left(\frac{\mu}{\beta}(R_0 - 1) - \frac{R_0}{\beta} \sum_{i=1}^{n} p_i \right) - I \right], I(0) = I_0$$

We will solve this for the steady-state in any case as it is an infinite horizon problem so there is no loss of generality by approximating the state dynamics by a a simple harmonic oscillator.

Open-loop solution

The current value hamiltonian:

$$\tilde{H} = r(1 - I) - \frac{c_i}{2} p_i^2 - \frac{b}{2} I^2 + \lambda\sigma \left[\left(\frac{\mu}{\beta}(R_0 - 1) - \frac{R_0}{\beta} \sum_{i=1}^{n} p_i \right) - I \right]$$

Pontryagin's maximum principle

$$\frac{\partial H}{\partial p_i} = -c_i p_i - \lambda\sigma \frac{R_0}{\beta} = 0$$

$$\dot{\lambda} - \rho\lambda = -\frac{\partial H}{\partial I} = r + bI + \lambda\sigma$$

Solution

After some algebra we get

$$p_i = \frac{(r + bI^{**})(\sigma R_0)}{c_i \beta(\sigma + \rho)}$$

where I^{**} is the steady-state level of prevalence corresponding to an open-loop Nash equilibrium.

Feedback (Markov Perfect) Nash equilibrium

Definition

Markov Perfect Nash Equilibrium A strategy profile $(p_1^*, \ldots, p_n^*)$ is an Markov Perfect Nash equilibrium if
$V^i(p_1^*, \ldots, p_n^*) \geq V^i(p_1^*, \ldots, , p_i, \ldots, p_n^*)$ and $p_i(t) = \psi(I, t), \forall i$.
These strategies are not history dependent in the sense that they do not depend on past play of the game (this is an active area of current research).

Feedback solution

$$\rho W^i - W^i_t = \max_{p_i(t)} \Big\{ r(1-I) - \frac{c_i}{2} p_i^2 - \frac{b}{2} I^2 +$$

$$W^i_I \left[\sigma \left[\left(\frac{\mu}{\beta}(R_0 - 1) - \frac{R_0}{\beta} \sum_{i=1}^{n} p_i \right) - I \right] \right] \Big\}$$

Evaluating the maximum we obtain
$p^*_i(t) = -\frac{W_I \sigma}{c_i} \frac{R_0}{\beta} = -\frac{(v^{i*} I + w^{i*})\sigma}{c_i} \frac{R_0}{\beta}$ which on substitution results in

$$\rho W - W_t = \Big\{ r(1-I) - \frac{c_i}{2} (\frac{W_I \sigma}{c_i} \frac{R_0}{\beta})^2 - \frac{b}{2} I^2 +$$

$$W_I \left[\sigma \left[\left(\frac{\mu}{\beta}(R_0 - 1) + \frac{R_0}{\beta} \sum_{i=1}^{n} \frac{W_I \sigma}{c_i} \frac{R_0}{\beta} \right) - I \right] \right] \Big\}$$

Solution

We conjecture the following solution
$W(t, I) = \frac{1}{2}v^i(t)I(t)^2 + w^i(t)I(t) + z^i(t)$
In the two-player case this leads to a system of equations of the following form:

$$A_1(v^1)^2 + B_1 v^1 + C_2(v^2)^2 + Dv^2 v^1 + E = 0$$

$$A_2(v^2)^2 + B_2 v^2 + C_1(v^1)^2 + Dv^2 v^1 + E = 0$$

along with two equations in w and two in z.
Giving a total of 6 polynomial equations. so far unable to obtain an analytical solution for these but a numerical solution is possible.

Numerical results

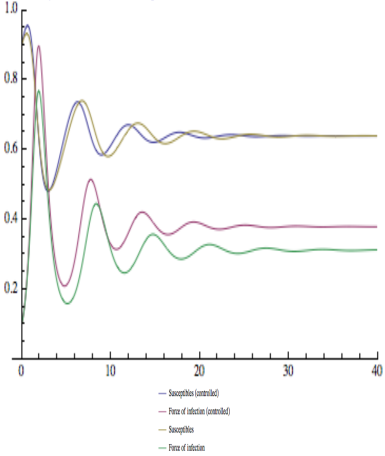
Table : Table of coefficients

	equilibrium 1		equilibrium 2	
	p_1	p_2	p_1	p_2
intercept	-0.0001025636	-0.000102318	-0.000102318	-0.0001025636
slope	968250	1936500	138321	138321

	equilibrium 3		equilibrium 4	
	p_1	p_2	p_1	p_2
intercept	-0.0001025636	-0.000102318	-0.00010356	-0.00010356
slope	0.403435	0.403435	1936500	968250

Controlled simulation of the original system

Dynamics induced by strategic behaviour



Transmission rate

Solution is of the form:

$$p_i^* = -\frac{v^{i*}I + w^{i*}}{c_i}\frac{R_0}{\beta} = -a + bI, i = 1, \ldots, n$$

$$\dot{S} = \mu - (\lambda + \mu)S - p = \mu - (\beta I + \mu)S - \sum p_i$$

$$\dot{S} = \mu - \beta SI - \mu S + \sum \frac{v^{i*}I + w^{i*}}{c_i}\frac{R_0}{\beta}$$

$$= \mu + (\sum_i \frac{v^{i*}R_0}{\beta c_i} - \beta S)I - \mu S + \sum \frac{w^{i*}}{c_i}\frac{R_0}{\beta}$$

so we obtain an affine transform of the transmission rate of the disease. Effectively, this lowers the disease transmission.

Risk aversion and the prevalence elasticity of prevention

Proposition

The prevalence elasticity of prevention is the negative of the rate of relative risk aversion.

Proof.

$W_I^i = v^{i*} I + w^{i*}$, differentiating this again we see that $W_{II}^i = v^{i*}$ substituting we get $W_I = W_{II} I + w^{i*}$ so that $w^{i*} = W_I - W_{II} I$. Substituting these into the prevalence elasticity of prevention gives

$$\frac{dp_i^*}{dI} \frac{I}{p_i^*} = \frac{W_{II}^i I}{W_I^i}$$

which is just the negative of the rate of relative risk aversion $-\frac{W_{II} I}{W_I}$

. □

Empirical estimates of risk aversion as a function of prevalence would allow us to solve this ordinary differential equation for the

Some caveats

- linear approximation imperfect only accurate close to steady-state
- I have not explored eradication conditions yet.
- Have not considered other disease dynamics

Conclusion and further work

- Differential games provide a useful framework for thinking about bioeconomic models of epidemiology
- Information structures need to be considered to properly account for behavioural feedbacks
- Further work needs to be done to explore issue of multiple equilibria and stability in LQ-games
- Further work needs to be done in exploring analytical solutions using methods from algebraic geometry.
- Other dynamic games such as stochastic games may be computationally more tractable although less intuitive (currently doing this in other work).

Thanks for listening!

Contact: Rodney.M.Beard@gmail.com
http://rbeard.sdf.org
IBAHCM
University of Glasgow
http://www.gla.ac.uk/researchinstitutes/bahcm/